

Prove that $\lim_{x \rightarrow 0} x^4 \tan^{-1} \frac{1}{x} = 0$.

SCORE: _____ / 5 PTS

①
$$\left| -\frac{\pi}{2} \leq \tan^{-1} \frac{1}{x} \leq \frac{\pi}{2} \right| \text{ FOR ALL } x \neq 0$$

①
$$\left| -\frac{\pi}{2}x^4 \leq x^4 \tan^{-1} \frac{1}{x} \leq \frac{\pi}{2}x^4 \right|$$

①
$$\left| \lim_{x \rightarrow 0} -\frac{\pi}{2}x^4 = \lim_{x \rightarrow 0} \frac{\pi}{2}x^4 = 0 \right|$$

① BY SQUEEZE THEOREM,
$$\left| \lim_{x \rightarrow 0} x^4 \tan^{-1} \frac{1}{x} = 0 \right| \text{ ①}$$

Sketch the graph of an example of a function that satisfies all the following conditions.

SCORE: _____ / 2 PTS

The domain of the function is $[-5, 3] \cup (3, 5]$

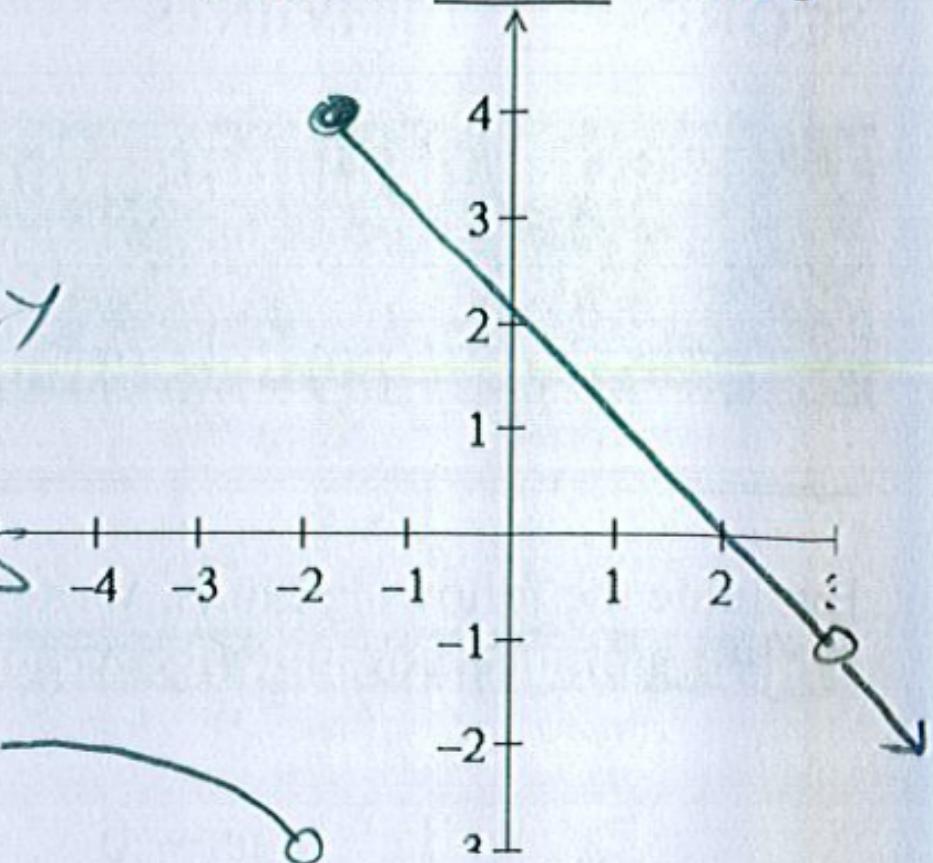
$$\lim_{x \rightarrow -2^+} f(x) = 4$$

$$\lim_{x \rightarrow -2^-} f(x) = -3$$

$$\lim_{x \rightarrow 3} f(x) = -1$$

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MANY
SOLUTIONS



The graph of f is shown on the right. Evaluate the following limits. Write "DNE" if a limit does not exist.

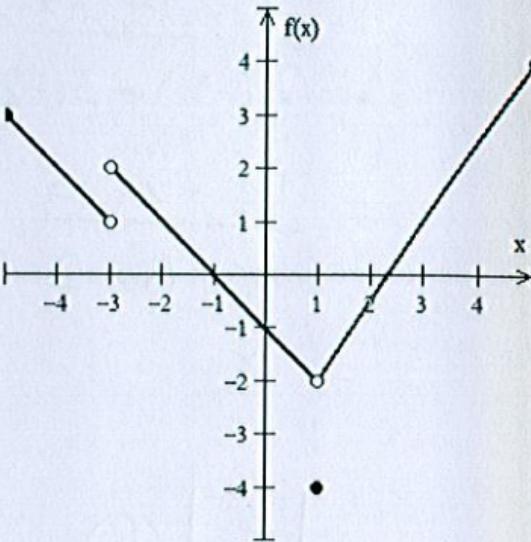
SCORE: _____ / 4 PTS

[a] $\lim_{x \rightarrow 1} \frac{x}{7 - [f(x)]^2}$ ← Show the proper use of limit laws to find your answer.

$$= \left| \frac{\lim_{x \rightarrow 1} x}{\lim_{x \rightarrow 1} 7 - \lim_{x \rightarrow 1} f(x) \lim_{x \rightarrow 1} f(x)} \right|$$
$$= \left| \frac{1}{7 - (-2)^2} \right| \textcircled{1}$$
$$= \left| \frac{1}{3} \right| \textcircled{2}$$

[b] $\lim_{x \rightarrow -3^-} f(x)$

$$= \boxed{1} \textcircled{1}$$



$$\lim_{x \rightarrow 3} \frac{\frac{4}{1-x} + 2}{1 - \frac{3}{x}} \quad \frac{0}{0} \text{ INDETERMINATE}$$

$$= \lim_{x \rightarrow 3} \left(\frac{4+2-2x}{1-x} \div \frac{x-3}{x} \right)$$

$$= \boxed{\lim_{x \rightarrow 3} \frac{-2}{1-x} \cdot \frac{x}{x-3}} \quad \textcircled{1}$$

$$= \boxed{\lim_{x \rightarrow 3} \frac{-2x}{1-x}} \quad \textcircled{1}$$

$$= \frac{-6}{-2}$$

$$= \boxed{3} \quad \textcircled{1}$$

$$\lim_{x \rightarrow 5} f(x) \text{ if } f(x) = \begin{cases} \sqrt{x+4}, & \text{if } x < 5 \\ 2, & \text{if } x = 5 \\ \frac{6}{x-3}, & \text{if } x > 5 \end{cases}$$

$$\lim_{x \rightarrow 5^-} f(x) = \boxed{\lim_{x \rightarrow 5^-} \sqrt{x+4} = \sqrt{9}} = 3$$

$$\lim_{x \rightarrow 5^+} f(x) = \boxed{\lim_{x \rightarrow 5^+} \frac{6}{x-3} = \frac{6}{2}} = 3$$

$$\boxed{\lim_{x \rightarrow 5} f(x) = 3}$$

$$\lim_{x \rightarrow -2} \frac{3x^2 - x - 6}{2x^2 - 3x - 2}$$

$$= \frac{3(-2)^2 - (-2) - 6}{2(-2)^2 - 3(-2) - 2}$$

$$= \frac{12 + 2 - 6}{8 + 6 - 2}$$

$$= \frac{8}{12}$$

$$= \boxed{\frac{2}{3}} \textcircled{1}$$

$$\lim_{x \rightarrow -4} \frac{1 - \sqrt{3x + 13}}{8 + 2x} \quad \frac{0}{0} \text{ INDETERMINATE}$$

$$= \lim_{x \rightarrow -4} \frac{(1 - \sqrt{3x + 13})(1 + \sqrt{3x + 13})}{(8 + 2x)(1 + \sqrt{3x + 13})}$$

$$= \boxed{\lim_{x \rightarrow -4} \frac{1 - (3x + 13)}{(8 + 2x)(1 + \sqrt{3x + 13})}}$$

$$= \lim_{x \rightarrow -4} \frac{\cancel{-3x - 12} - 3}{\cancel{(8 + 2x)}^2 (1 + \sqrt{3x + 13})} \quad \textcircled{1}$$

$$= \boxed{\lim_{x \rightarrow -4} \frac{-3}{2(1 + \sqrt{3x + 13})}} \quad \textcircled{1}$$

$$= \frac{-3}{2 \cdot 2} = \boxed{-\frac{3}{4}} \quad \textcircled{1}$$